CHAPTER 3

The Cellular Concept—
System Design Fundamentals

The design objective of early mobile radio systems was to achieve a large coverage area by using a single, high powered transmitter with an antenna mounted on a tall tower. While this approach achieved very good coverage, it also meant that it was impossible to reuse those same frequencies throughout the system, since any attempts to achieve frequency reuse would result in interference. For example, the Bell mobile system in New York City in the 1970s could only support a maximum of twelve simultaneous calls over a thousand square miles [Cal88]. Faced with the fact that government regulatory agencies could not make spectrum allocations in proportion to the increasing demand for mobile services, it became imperative to restructure the radio telephone system to achieve high capacity with limited radio spectrum while at the same time covering very large areas.

3.1 Introduction

The cellular concept was a major breakthrough in solving the problem of spectral congestion and user capacity. It offered very high capacity in a limited spectrum allocation without any major technological changes. The cellular concept is a system-level idea which calls for replacing a single, high power transmitter (large cell) with many low power transmitters (small cells), each providing coverage to only a small portion of the service area. Each base station is allocated a portion of the total number of channels available to the entire system, and nearby base stations are assigned different groups of channels so that all the available channels are assigned to a relatively small number of neighboring base stations. Neighboring base stations are assigned different groups of channels so that the interference between base stations (and the mobile users under their control) is minimized. By systematically spacing base stations and their channel
groups throughout a market, the available channels are distributed throughout the geographic region and may be reused as many times as necessary so long as the interference between co-channel stations is kept below acceptable levels.

As the demand for service increases (i.e., as more channels are needed within a particular market), the number of base stations may be increased (along with a corresponding decrease in transmitter power to avoid added interference), thereby providing additional radio capacity with no additional increase in radio spectrum. This fundamental principle is the foundation for all modern wireless communication systems, since it enables a fixed number of channels to serve an arbitrarily large number of subscribers by reusing the channels throughout the coverage region. Furthermore, the cellular concept allows every piece of subscriber equipment within a country or continent to be manufactured with the same set of channels so that any mobile may be used anywhere within the region.

3.2 Frequency Reuse

Cellular radio systems rely on an intelligent allocation and reuse of channels throughout a coverage region [Oet83]. Each cellular base station is allocated a group of radio channels to be used within a small geographic area called a cell. Base stations in adjacent cells are assigned channel groups which contain completely different channels than neighboring cells. The base station antennas are designed to achieve the desired coverage within the particular cell. By limiting the coverage area to within the boundaries of a cell, the same group of channels may be used to cover different cells that are separated from one another by distances large enough to keep interference levels within tolerable limits. The design process of selecting and allocating channel groups for all of the cellular base stations within a system is called frequency reuse or frequency planning [Mac79].

Figure 3.1 illustrates the concept of cellular frequency reuse, where cells labeled with the same letter use the same group of channels. The frequency reuse plan is overlaid upon a map to indicate where different frequency channels are used. The hexagonal cell shape shown in Figure 3.1 is conceptual and is a simplistic model of the radio coverage for each base station, but it has been universally adopted since the hexagon permits easy and manageable analysis of a cellular system. The actual radio coverage of a cell is known as the footprint and is determined from field measurements or propagation prediction models. Although the real footprint is amorphous in nature, a regular cell shape is needed for systematic system design and adaptation for future growth. While it might seem natural to choose a circle to represent the coverage area of a base station, adjacent circles cannot be overlaid upon a map without leaving gaps or creating overlapping regions. Thus, when considering geometric shapes which cover an entire region without overlap and with equal area, there are three sensible choices—a square, an equilateral triangle, and a hexagon. A cell must be designed to serve the weakest mobiles within the footprint, and these are typically located at the edge of the cell. For a given distance between the center of a polygon and its farthest perimeter points, the hexagon has the largest area of the three. Thus, by using the hexagon geometry, the fewest number of cells can cover a geographic region, and the hexagon closely approximates a circular radiation pattern which would occur for an omnidirectional base
station antenna and free space propagation. Of course, the actual cellular footprint is determined by
the contour in which a given transmitter serves the mobiles successfully.

When using hexagons to model coverage areas, base station transmitters are depicted as either
being in the center of the cell (center-excited cells) or on three of the six cell vertices (edge-excited
cells). Normally, omnidirectional antennas are used in center-excited cells and sectored directional
antennas are used in corner-excited cells. Practical considerations usually do not allow base stations
to be placed exactly as they appear in the hexagonal layout. Most system designs permit a base sta-
tion to be positioned up to one-fourth the cell radius away from the ideal location.

To understand the frequency reuse concept, consider a cellular system which has a total of
$S$ duplex channels available for use. If each cell is allocated a group of $k$ channels ($k < S$), and if the
$S$ channels are divided among $N$ cells into unique and disjoint channel groups which each have
the same number of channels, the total number of available radio channels can be expressed as

$$S = kN \tag{3.1}$$

The $N$ cells which collectively use the complete set of available frequencies is called a
cluster. If a cluster is replicated $M$ times within the system, the total number of duplex channels,
$C$, can be used as a measure of capacity and is given by

$$C = MkN = MS \tag{3.2}$$
As seen from Equation (3.2), the capacity of a cellular system is directly proportional to the number of times a cluster is replicated in a fixed service area. The factor $N$ is called the **cluster size** and is typically equal to 4, 7, or 12. If the cluster size $N$ is reduced while the cell size is kept constant, more clusters are required to cover a given area, and hence more capacity (a larger value of $C$) is achieved. A large cluster size indicates that the ratio between the cell radius and the distance between co-channel cells is small. Conversely, a small cluster size indicates that co-channel cells are located much closer together. The value for $N$ is a function of how much interference a mobile or base station can tolerate while maintaining a sufficient quality of communications. From a design viewpoint, the smallest possible value of $N$ is desirable in order to maximize capacity over a given coverage area (i.e., to maximize $C$ in Equation (3.2)). The **frequency reuse factor** of a cellular system is given by $1/N$, since each cell within a cluster is only assigned $1/N$ of the total available channels in the system.

Due to the fact that the hexagonal geometry of Figure 3.1 has exactly six equidistant neighbors and that the lines joining the centers of any cell and each of its neighbors are separated by multiples of 60 degrees, there are only certain cluster sizes and cell layouts which are possible [Mac79]. In order to tessellate—to connect without gaps between adjacent cells—the geometry of hexagons is such that the number of cells per cluster, $N$, can only have values which satisfy Equation (3.3).

$$N = i^2 + ij + j^2$$  \hspace{1cm} (3.3)

where $i$ and $j$ are non-negative integers. To find the nearest co-channel neighbors of a particular cell, one must do the following: (1) move $i$ cells along any chain of hexagons and then (2) turn 60 degrees counter-clockwise and move $j$ cells. This is illustrated in Figure 3.2 for $i = 3$ and $j = 2$ (example, $N = 19$).

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**Figure 3.2** Method of locating co-channel cells in a cellular system. In this example, $N = 19$ (i.e., $i = 3$, $j = 2$). (Adapted from [Oet83] © IEEE.)
Example 3.1

If a total of 33 MHz of bandwidth is allocated to a particular FDD cellular telephone system which uses two 25 kHz simplex channels to provide full duplex voice and control channels, compute the number of channels available per cell if a system uses (a) four-cell reuse, (b) seven-cell reuse, and (c) 12-cell reuse. If 1 MHz of the allocated spectrum is dedicated to control channels, determine an equitable distribution of control channels and voice channels in each cell for each of the three systems.

Solution

Given:
Total bandwidth = 33 MHz
Channel bandwidth = 25 kHz × 2 simplex channels = 50 kHz/duplex channel
Total available channels = 33,000/50 = 660 channels
(a) For \( N = 4 \),
   total number of channels available per cell = \( \frac{660}{4} \approx 165 \) channels.
(b) For \( N = 7 \),
   total number of channels available per cell = \( \frac{660}{7} \approx 95 \) channels.
(c) For \( N = 12 \),
   total number of channels available per cell = \( \frac{660}{12} \approx 55 \) channels.

A 1 MHz spectrum for control channels implies that there are \( \frac{1000}{50} = 20 \) control channels out of the 660 channels available. To evenly distribute the control and voice channels, simply allocate the same number of voice channels in each cell wherever possible. Here, the 660 channels must be evenly distributed to each cell within the cluster. In practice, only the 640 voice channels would be allocated, since the control channels are allocated separately as 1 per cell.

(a) For \( N = 4 \), we can have five control channels and 160 voice channels per cell. In practice, however, each cell only needs a single control channel (the control channels have a greater reuse distance than the voice channels). Thus, one control channel and 160 voice channels would be assigned to each cell.

(b) For \( N = 7 \), four cells with three control channels and 92 voice channels, two cells with three control channels and 90 voice channels, and one cell with two control channels and 92 voice channels could be allocated. In practice, however, each cell would have one control channel, four cells would have 91 voice channels, and three cells would have 92 voice channels.

(c) For \( N = 12 \), we can have eight cells with two control channels and 53 voice channels, and four cells with one control channel and 54 voice channels each. In an actual system, each cell would have one control channel, eight cells would have 53 voice channels, and four cells would have 54 voice channels.
3.3 Channel Assignment Strategies

For efficient utilization of the radio spectrum, a frequency reuse scheme that is consistent with the objectives of increasing capacity and minimizing interference is required. A variety of channel assignment strategies have been developed to achieve these objectives. Channel assignment strategies can be classified as either fixed or dynamic. The choice of channel assignment strategy impacts the performance of the system, particularly as to how calls are managed when a mobile user is handed off from one cell to another [Tek91], [LiC93], [Sun94], [Rap93b].

In a fixed channel assignment strategy, each cell is allocated a predetermined set of voice channels. Any call attempt within the cell can only be served by the unused channels in that particular cell. If all the channels in that cell are occupied, the call is blocked and the subscriber does not receive service. Several variations of the fixed assignment strategy exist. In one approach, called the borrowing strategy, a cell is allowed to borrow channels from a neighboring cell if all of its own channels are already occupied. The mobile switching center (MSC) supervises such borrowing procedures and ensures that the borrowing of a channel does not disrupt or interfere with any of the calls in progress in the donor cell.

In a dynamic channel assignment strategy, voice channels are not allocated to different cells permanently. Instead, each time a call request is made, the serving base station requests a channel from the MSC. The switch then allocates a channel to the requested cell following an algorithm that takes into account the likelihood of future blocking within the cell, the frequency of use of the candidate channel, the reuse distance of the channel, and other cost functions. Accordingly, the MSC only allocates a given frequency if that frequency is not presently in use in the cell or any other cell which falls within the minimum restricted distance of frequency reuse to avoid co-channel interference. Dynamic channel assignment reduce the likelihood of blocking, which increases the trunking capacity of the system, since all the available channels in a market are accessible to all of the cells. Dynamic channel assignment strategies require the MSC to collect real-time data on channel occupancy, traffic distribution, and radio signal strength indications (RSSI) of all channels on a continuous basis. This increases the storage and computational load on the system but provides the advantage of increased channel utilization and decreased probability of a blocked call.

3.4 Handoff Strategies

When a mobile moves into a different cell while a conversation is in progress, the MSC automatically transfers the call to a new channel belonging to the new base station. This handoff operation not only involves identifying a new base station, but also requires that the voice and control signals be allocated to channels associated with the new base station.

Processing handoffs is an important task in any cellular radio system. Many handoff strategies prioritize handoff requests over call initiation requests when allocating unused channels in a cell site. Handoffs must be performed successfully and as infrequently as possible, and be imperceptible to the users. In order to meet these requirements, system designers must specify an optimum signal level at which to initiate a handoff. Once a particular signal level is specified as the minimum usable
signal for acceptable voice quality at the base station receiver (normally taken as between \(-90\) dBm and \(-100\) dBm), a slightly stronger signal level is used as a threshold at which a handoff is made. This margin, given by \(\Delta = P_r,\text{handoff} - P_r,\text{minimum usable}\), cannot be too large or too small. If \(\Delta\) is too large, unnecessary handoffs which burden the MSC may occur, and if \(\Delta\) is too small, there may be insufficient time to complete a handoff before a call is lost due to weak signal conditions. Therefore, \(\Delta\) is chosen carefully to meet these conflicting requirements. Figure 3.3 illustrates a handoff situation. Figure 3.3(a) demonstrates the case where a handoff is not made and the signal drops below the minimum acceptable level to keep the channel active. This dropped call event can happen when there is an excessive delay by the MSC in assigning a handoff or when the threshold \(\Delta\) is set too small for the handoff time in the system. Excessive delays may occur during high traffic conditions due to computational loading at the MSC or due to the fact that no channels are available on any of the nearby base stations (thus forcing the MSC to wait until a channel in a nearby cell becomes free).

Figure 3.3 Illustration of a handoff scenario at cell boundary.
In deciding when to handoff, it is important to ensure that the drop in the measured signal level is not due to momentary fading and that the mobile is actually moving away from the serving base station. In order to ensure this, the base station monitors the signal level for a certain period of time before a handoff is initiated. This running average measurement of signal strength should be optimized so that unnecessary handoffs are avoided, while ensuring that necessary handoffs are completed before a call is terminated due to poor signal level. The length of time needed to decide if a handoff is necessary depends on the speed at which the vehicle is moving. If the slope of the short-term average received signal level in a given time interval is steep, the handoff should be made quickly. Information about the vehicle speed, which can be useful in handoff decisions, can also be computed from the statistics of the received short-term fading signal at the base station.

The time over which a call may be maintained within a cell, without handoff, is called the *dwell time* [Rap93b]. The dwell time of a particular user is governed by a number of factors, including propagation, interference, distance between the subscriber and the base station, and other time varying effects. Chapter 5 shows that even when a mobile user is stationary, ambient motion in the vicinity of the base station and the mobile can produce fading; thus, even a stationary subscriber may have a random and finite dwell time. Analysis in [Rap93b] indicates that the statistics of dwell time vary greatly, depending on the speed of the user and the type of radio coverage. For example, in mature cells which provide coverage for vehicular highway users, most users tend to have a relatively constant speed and travel along fixed and well-defined paths with good radio coverage. In such instances, the dwell time for an arbitrary user is a random variable with a distribution that is highly concentrated about the mean dwell time. On the other hand, for users in dense, cluttered microcell environments, there is typically a large variation of dwell time about the mean, and the dwell times are typically shorter than the cell geometry would otherwise suggest. It is apparent that the statistics of dwell time are important in the practical design of handoff algorithms [LiC93], [Sun94], [Rap93b].

In first generation analog cellular systems, signal strength measurements are made by the base stations and supervised by the MSC. Each base station constantly monitors the signal strengths of all of its reverse voice channels to determine the relative location of each mobile user with respect to the base station tower. In addition to measuring the RSSI of calls in progress within the cell, a spare receiver in each base station, called the locator receiver, is used to scan and determine signal strengths of mobile users which are in neighboring cells. The *locator receiver* is controlled by the MSC and is used to monitor the signal strength of users in neighboring cells which appear to be in need of handoff and reports all RSSI values to the MSC. Based on the locator receiver signal strength information from each base station, the MSC decides if a handoff is necessary or not.

In today’s second generation systems, handoff decisions are *mobile assisted*. In *mobile assisted handoff* (MAHO), every mobile station measures the received power from surrounding base stations and continually reports the results of these measurements to the serving base station. A handoff is initiated when the power received from the base station of a neighboring cell begins to exceed the power received from the current base station by a certain level or for a certain period of
time. The MAHO method enables the call to be handed over between base stations at a much faster rate than in first generation analog systems since the handoff measurements are made by each mobile, and the MSC no longer constantly monitors signal strengths. MAHO is particularly suited for microcellular environments where handoffs are more frequent.

During the course of a call, if a mobile moves from one cellular system to a different cellular system controlled by a different MSC, an intersystem handoff becomes necessary. An MSC engages in an intersystem handoff when a mobile signal becomes weak in a given cell and the MSC cannot find another cell within its system to which it can transfer the call in progress. There are many issues that must be addressed when implementing an intersystem handoff. For instance, a local call may become a long-distance call as the mobile moves out of its home system and becomes a roamer in a neighboring system. Also, compatibility between the two MSCs must be determined before implementing an intersystem handoff. Chapter 10 demonstrates how intersystem handoffs are implemented in practice.

Different systems have different policies and methods for managing handoff requests. Some systems handle handoff requests in the same way they handle originating calls. In such systems, the probability that a handoff request will not be served by a new base station is equal to the blocking probability of incoming calls. However, from the user’s point of view, having a call abruptly terminated while in the middle of a conversation is more annoying than being blocked occasionally on a new call attempt. To improve the quality of service as perceived by the users, various methods have been devised to prioritize handoff requests over call initiation requests when allocating voice channels.

### 3.4.1 Prioritizing Handoffs

One method for giving priority to handoffs is called the guard channel concept, whereby a fraction of the total available channels in a cell is reserved exclusively for handoff requests from ongoing calls which may be handed off into the cell. This method has the disadvantage of reducing the total carried traffic, as fewer channels are allocated to originating calls. Guard channels, however, offer efficient spectrum utilization when dynamic channel assignment strategies, which minimize the number of required guard channels by efficient demand-based allocation, are used.

Queuing of handoff requests is another method to decrease the probability of forced termination of a call due to lack of available channels. There is a tradeoff between the decrease in probability of forced termination and total carried traffic. Queuing of handoffs is possible due to the fact that there is a finite time interval between the time the received signal level drops below the handoff threshold and the time the call is terminated due to insufficient signal level. The delay time and size of the queue is determined from the traffic pattern of the particular service area. It should be noted that queuing does not guarantee a zero probability of forced termination, since large delays will cause the received signal level to drop below the minimum required level to maintain communication and hence lead to forced termination.
3.4.2 Practical Handoff Considerations

In practical cellular systems, several problems arise when attempting to design for a wide range of mobile velocities. High speed vehicles pass through the coverage region of a cell within a matter of seconds, whereas pedestrian users may never need a handoff during a call. Particularly with the addition of microcells to provide capacity, the MSC can quickly become burdened if high speed users are constantly being passed between very small cells. Several schemes have been devised to handle the simultaneous traffic of high speed and low speed users while minimizing the handoff intervention from the MSC. Another practical limitation is the ability to obtain new cell sites.

Although the cellular concept clearly provides additional capacity through the addition of cell sites, in practice it is difficult for cellular service providers to obtain new physical cell site locations in urban areas. Zoning laws, ordinances, and other nontechnical barriers often make it more attractive for a cellular provider to install additional channels and base stations at the same physical location of an existing cell, rather than find new site locations. By using different antenna heights (often on the same building or tower) and different power levels, it is possible to provide “large” and “small” cells which are co-located at a single location. This technique is called the umbrella cell approach and is used to provide large area coverage to high speed users while providing small area coverage to users traveling at low speeds. Figure 3.4 illustrates an umbrella cell which is co-located with some smaller microcells. The umbrella cell approach ensures that the number of handoffs is minimized for high speed users and provides additional microcell channels for pedestrian users. The speed of each user may be estimated by the base station or MSC by evaluating how rapidly the short-term average signal strength on the RVC changes over time, or more sophisticated algorithms may be used to evaluate and partition users [LiC93]. If a high speed user in the large umbrella cell is approaching the base station, and its velocity is rapidly decreasing, the base station may decide to hand the user into the co-located microcell, without MSC intervention.

Another practical handoff problem in microcell systems is known as cell dragging. Cell dragging results from pedestrian users that provide a very strong signal to the base station. Such a situation occurs in an urban environment when there is a line-of-sight (LOS) radio path between the subscriber and the base station. As the user travels away from the base station at a very slow speed, the average signal strength does not decay rapidly. Even when the user has traveled well beyond the designed range of the cell, the received signal at the base station may be above the handoff threshold, thus a handoff may not be made. This creates a potential interference and traffic management problem, since the user has meanwhile traveled deep within a neighboring cell. To solve the cell dragging problem, handoff thresholds and radio coverage parameters must be adjusted carefully.

In first generation analog cellular systems, the typical time to make a handoff, once the signal level is deemed to be below the handoff threshold, is about 10 seconds. This requires that the value for $\Delta$ be on the order of 6 dB to 12 dB. In digital cellular systems such as GSM, the mobile assists with the handoff procedure by determining the best handoff candidates, and the handoff, once the decision is made, typically requires only 1 or 2 seconds. Consequently, $\Delta$ is
usually between 0 dB and 6 dB in modern cellular systems. The faster handoff process supports a much greater range of options for handling high speed and low speed users and provides the MSC with substantial time to “rescue” a call that is in need of handoff.

Another feature of newer cellular systems is the ability to make handoff decisions based on a wide range of metrics other than signal strength. The co-channel and adjacent channel interference levels may be measured at the base station or the mobile, and this information may be used with conventional signal strength data to provide a multi-dimensional algorithm for determining when a handoff is needed.

The IS-95 code division multiple access (CDMA) spread spectrum cellular system described in Chapter 11 and in [Lib99], [Kim00], and [Gar99], provides a unique handoff capability that cannot be provided with other wireless systems. Unlike channelized wireless systems that assign different radio channels during a handoff (called a hard handoff), spread spectrum mobiles share the same channel in every cell. Thus, the term handoff does not mean a physical change in the assigned channel, but rather that a different base station handles the radio communication task. By simultaneously evaluating the received signals from a single subscriber at several neighboring base stations, the MSC may actually decide which version of the user’s signal is best at any moment in time. This technique exploits macroscopic space diversity provided by the different physical locations of the base stations and allows the MSC to make a “soft” decision as to which version of the user’s signal to pass along to the PSTN at any instance [Pad94]. The ability to select between the instantaneous received signals from a variety of base stations is called soft handoff.

3.5 Interference and System Capacity

Interference is the major limiting factor in the performance of cellular radio systems. Sources of interference include another mobile in the same cell, a call in progress in a neighboring cell, other base stations operating in the same frequency band, or any noncellular system which inadvertently
leaks energy into the cellular frequency band. Interference on voice channels causes cross talk, where the subscriber hears interference in the background due to an undesired transmission. On control channels, interference leads to missed and blocked calls due to errors in the digital signaling. Interference is more severe in urban areas, due to the greater RF noise floor and the large number of base stations and mobiles. Interference has been recognized as a major bottleneck in increasing capacity and is often responsible for dropped calls. The two major types of system-generated cellular interference are co-channel interference and adjacent channel interference. Even though interfering signals are often generated within the cellular system, they are difficult to control in practice (due to random propagation effects). Even more difficult to control is interference due to out-of-band users, which arises without warning due to front end overload of subscriber equipment or intermittent intermodulation products. In practice, the transmitters from competing cellular carriers are often a significant source of out-of-band interference, since competitors often locate their base stations in close proximity to one another in order to provide comparable coverage to customers.

### 3.5.1 Co-channel Interference and System Capacity

Frequency reuse implies that in a given coverage area there are several cells that use the same set of frequencies. These cells are called co-channel cells, and the interference between signals from these cells is called co-channel interference. Unlike thermal noise which can be overcome by increasing the signal-to-noise ratio (SNR), co-channel interference cannot be combated by simply increasing the carrier power of a transmitter. This is because an increase in carrier transmit power increases the interference to neighboring co-channel cells. To reduce co-channel interference, co-channel cells must be physically separated by a minimum distance to provide sufficient isolation due to propagation.

When the size of each cell is approximately the same and the base stations transmit the same power, the co-channel interference ratio is independent of the transmitted power and becomes a function of the radius of the cell \( R \) and the distance between centers of the nearest co-channel cells \( D \). By increasing the ratio of \( D/R \), the spatial separation between co-channel cells relative to the coverage distance of a cell is increased. Thus, interference is reduced from improved isolation of RF energy from the co-channel cell. The parameter \( Q \), called the co-channel reuse ratio, is related to the cluster size (see Table 3.1 and Equation (3.3)). For a hexagonal geometry

\[
Q = \frac{D}{R} = \sqrt[3]{3N}
\]

A small value of \( Q \) provides larger capacity since the cluster size \( N \) is small, whereas a large value of \( Q \) improves the transmission quality, due to a smaller level of co-channel interference. A trade-off must be made between these two objectives in actual cellular design.
Let $i_0$ be the number of co-channel interfering cells. Then, the signal-to-interference ratio ($S/I$ or $SIR$) for a mobile receiver which monitors a forward channel can be expressed as

$$\frac{S}{I} = \frac{S}{\sum_{i=1}^{i_0} I_i} \tag{3.5}$$

where $S$ is the desired signal power from the desired base station and $I_i$ is the interference power caused by the $i$th interfering co-channel cell base station. If the signal levels of co-channel cells are known, then the $S/I$ ratio for the forward link can be found using Equation (3.5).

Propagation measurements in a mobile radio channel show that the average received signal strength at any point decays as a power law of the distance of separation between a transmitter and receiver. The average received power $P_r$ at a distance $d$ from the transmitting antenna is approximated by

$$P_r = P_0 \left( \frac{d}{d_0} \right)^{-n} \tag{3.6}$$

or

$$P_r(\text{dBm}) = P_0(\text{dBm}) - 10n\log\left( \frac{d}{d_0} \right) \tag{3.7}$$

where $P_0$ is the power received at a close-in reference point in the far field region of the antenna at a small distance $d_0$ from the transmitting antenna and $n$ is the path loss exponent. Now consider the forward link where the desired signal is the serving base station and where the interference is due to co-channel base stations. If $D_i$ is the distance of the $i$th interferer from the mobile, the received power at a given mobile due to the $i$th interfering cell will be proportional to $(D_i)^{-n}$. The path loss exponent typically ranges between two and four in urban cellular systems [Rap92b].

### Table 3.1  Co-channel Reuse Ratio for Some Values of $N$

<table>
<thead>
<tr>
<th>Cluster Size ($N$)</th>
<th>Co-channel Reuse Ratio ($Q$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i = 1, j = 1$</td>
<td>$3$</td>
</tr>
<tr>
<td>$i = 1, j = 2$</td>
<td>$7$</td>
</tr>
<tr>
<td>$i = 2, j = 2$</td>
<td>$12$</td>
</tr>
<tr>
<td>$i = 1, j = 3$</td>
<td>$13$</td>
</tr>
</tbody>
</table>

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When the transmit power of each base station is equal and the path loss exponent is the same throughout the coverage area, $S/I$ for a mobile can be approximated as

$$\frac{S}{I} = \frac{R^{-n}}{i_0} \sum_{j=1}^{i_0} (D_j)^{-n}$$  \hspace{1cm} (3.8)

Considering only the first layer of interfering cells, if all the interfering base stations are equidistant from the desired base station and if this distance is equal to the distance $D$ between cell centers, then Equation (3.8) simplifies to

$$\frac{S}{I} = \frac{(D/R)^n}{i_0} = \frac{(\sqrt{3}N)^n}{i_0}$$  \hspace{1cm} (3.9)

Equation (3.9) relates $S/I$ to the cluster size $N$, which in turn determines the overall capacity of the system from Equation (3.2). For example, assume that the six closest cells are close enough to create significant interference and that they are all approximately equidistant from the desired base station. For the U.S. AMPS cellular system which uses FM and 30 kHz channels, subjective tests indicate that sufficient voice quality is provided when $S/I$ is greater than or equal to 18 dB. Using Equation (3.9), it can be shown in order to meet this requirement, the cluster size $N$ should be at least 6.49, assuming a path loss exponent $n = 4$. Thus a minimum cluster size of seven is required to meet an $S/I$ requirement of 18 dB. It should be noted that Equation (3.9) is based on the hexagonal cell geometry where all the interfering cells are equidistant from the base station receiver, and hence provides an optimistic result in many cases. For some frequency reuse plans (e.g., $N = 4$), the closest interfering cells vary widely in their distances from the desired cell.

Using an exact cell geometry layout, it can be shown for a seven-cell cluster, with the mobile unit at the cell boundary, the mobile is a distance $D - R$ from the two nearest co-channel interfering cells and is exactly $D + R/2$, $D$, $D - R/2$, and $D + R$ from the other interfering cells in the first tier, as shown rigorously in [Lee86]. Using the approximate geometry shown in Figure 3.5, Equation (3.8), and assuming $n = 4$, the signal-to-interference ratio for the worst case can be closely approximated as (an exact expression is worked out by Jacobsmeyer [Jac94])

$$\frac{S}{I} = \frac{R^4}{2(D - R)^4 + 2(D + R)^4 + 2D^4}$$  \hspace{1cm} (3.10)

Equation (3.10) can be rewritten in terms of the co-channel reuse ratio $Q$, as

$$\frac{S}{I} = \frac{1}{2(Q - 1)^4 + 2(Q + 1)^4 + 2Q^4}$$  \hspace{1cm} (3.11)
For $N = 7$, the co-channel reuse ratio $Q$ is 4.6, and the worst case $S/I$ is approximated as 49.56 (17 dB) using Equation (3.11), whereas an exact solution using Equation (3.8) yields 17.8 dB [Jac94]. Hence for a seven-cell cluster, the $S/I$ ratio is slightly less than 18 dB for the worst case. To design the cellular system for proper performance in the worst case, it would be necessary to increase $N$ to the next largest size, which from Equation (3.3) is found to be 12 (corresponding to $i = j = 2$). This obviously entails a significant decrease in capacity, since 12-cell reuse offers a spectrum utilization of $1/12$ within each cell, whereas seven-cell reuse offers a spectrum utilization of $1/7$. In practice, a capacity reduction of $7/12$ would not be tolerable to accommodate for the worst case situation which rarely occurs. From the above discussion, it is clear that co-channel interference determines link performance, which in turn dictates the frequency reuse plan and the overall capacity of cellular systems.

Figure 3.5  Illustration of the first tier of co-channel cells for a cluster size of $N = 7$. An approximation of the exact geometry is shown here, whereas the exact geometry is given in [Lee86]. When the mobile is at the cell boundary (point X), it experiences worst case co-channel interference on the forward channel. The marked distances between the mobile and different co-channel cells are based on approximations made for easy analysis.
Example 3.2
If a signal-to-interference ratio of 15 dB is required for satisfactory forward channel performance of a cellular system, what is the frequency reuse factor and cluster size that should be used for maximum capacity if the path loss exponent is (a) $n = 4$, (b) $n = 3$? Assume that there are six co-channel cells in the first tier, and all of them are at the same distance from the mobile. Use suitable approximations.

Solution
(a) $n = 4$
First, let us consider a seven-cell reuse pattern.
Using Equation (3.4), the co-channel reuse ratio $D/R = 4.583$.
Using Equation (3.9), the signal-to-noise interference ratio is given by
$$S/I = \frac{1}{6} \times (4.583)^4 = 75.3 = 18.66 \text{ dB}$$
Since this is greater than the minimum required $S/I$, $N = 7$ can be used.

(b) $n = 3$
First, let us consider a seven-cell reuse pattern.
Using Equation (3.9), the signal-to-interference ratio is given by
$$S/I = \frac{1}{6} \times (4.583)^3 = 16.04 = 12.05 \text{ dB}$$
Since this is less than the minimum required $S/I$, we need to use a larger $N$.
Using Equation (3.3), the next possible value of $N$ is 12, $(i = j = 2)$.
The corresponding co-channel ratio is given by Equation (3.4) as
$$D/R = 6.0$$
Using Equation (3.3), the signal-to-interference ratio is given by
$$S/I = \frac{1}{6} \times (6)^3 = 36 = 15.56 \text{ dB}$$
Since this is greater than the minimum required $S/I$, $N = 12$ is used.

3.5.2 Channel Planning for Wireless Systems
Judiciously assigning the appropriate radio channels to each base station is an important process that is much more difficult in practice than in theory. While Equation (3.9) is a valuable rule of thumb for determining the appropriate frequency reuse ratio (or cluster size) and the appropriate separation between adjacent co-channel cells, the wireless engineer must deal with the real-world difficulties of radio propagation and imperfect coverage regions of each cell. Cellular systems, in practice, seldom obey the homogenous propagation path loss assumption of Equation (3.9).

Generally, the available mobile radio spectrum is divided into channels, which are part of an air interface standard that is used throughout a country or continent. These channels generally are made up of control channels (vital for initiating, requesting, or paging a call), and voice channels (dedicated to carrying revenue-generating traffic). Typically, about 5% of the entire mobile spectrum is devoted to control channels, which carry data messages that are very brief and bursty in nature, while the remaining 95% of the spectrum is dedicated to voice channels. Channels may be assigned
by the wireless carrier in any manner it chooses, since each market may have its own particular
propagation conditions or particular services it wishes to offer and may wish to adopt its own partic-
ular frequency reuse scheme that fits its geographic conditions or air interface technology choice.
However, in practical systems, the air interface standard ensures a distinction between voice and
control channels, and thus control channels are generally not allowed to be used as voice channels,
and vice versa. Furthermore, since control channels are vital in the successful launch of any call, the
frequency reuse strategy applied to control channels is different and generally more conservative
(e.g., is afforded greater S/I protection) than for the voice channels. This can be seen in Example 3.3,
where the control channels are allocated using 21-cell reuse, whereas voice channels are assigned
using seven-cell reuse. Typically, the control channels are able to handle a great deal of data such
that only one control channel is needed within a cell. As described in Section 3.7.2, sectoring is
often used to improve the signal-to-interference ratio which may lead to a smaller cluster size, and in
such cases, only a single control channel is assigned to an individual sector of a cell.

One of the key features of CDMA systems is that the cluster size is \( N = 1 \), and frequency
planning is not nearly as difficult as for TDMA or first generation cellular systems [Lib99]. Still,
however, propagation considerations require most practical CDMA systems to use some sort of
limited frequency reuse where propagation conditions are particularly ill-behaved in a particular
market. For example, in the vicinity of bodies of water, interfering cells on the same channel as
the desired serving cell can create interference overload that exceeds the dynamic range of
CDMA power control capabilities, leading to dropped calls. In such instances, the most popular
approach is to use what is called \( f_1/f_2 \) cell planning, where nearest neighbor cells use radio chan-
nels that are different from its closest neighbor in particular locations. Such frequency planning
requires CDMA phones to make hard handoffs, just as TDMA and FDMA phones do.

In CDMA, a single 1.25 MHz radio channel carries the simultaneous transmissions of the
single control channel with up to 64 simultaneous voice channels. Thus, unlike in 30 kHz IS-136 or
200 kHz GSM TDMA systems, where the coverage region and interference levels are well defined
when specific radio channels are in use, the CDMA system instead has a dynamic, time varying
coverage region which varies depending on the instantaneous number of users on the CDMA radio
channel. This effect, known as a breathing cell, requires the wireless engineer to carefully plan the
coverage and signal levels for the best and worst cases for serving cells as well as nearest neighbor
cells, from both a coverage and interference standpoint. The breathing cell phenomenon can lead to
abrupt dropped calls resulting from abrupt coverage changes simply due to an increase in the
number of users on a serving CDMA base station. Thus, instead of having to make careful decisions
about the channel assignment schemes for each cellular base station, CDMA engineers must instead
make difficult decisions about the power levels and thresholds assigned to control channels, voice
channels, and how these levels and thresholds should be adjusted for changing traffic intensity. Also,
threshold levels for CDMA handoffs, in both the soft handoff case and hard handoff case, must be
planned and often measured carefully before turning up service. In fact, the \( f_1/f_2 \) cell planning issue
led to the development of TSB-74, which added hard-handoff capabilities between different CDMA
radio channels to the original IS-95 CDMA specification described in Chapter 11.
3.5.3 Adjacent Channel Interference

Interference resulting from signals which are adjacent in frequency to the desired signal is called adjacent channel interference. Adjacent channel interference results from imperfect receiver filters which allow nearby frequencies to leak into the passband. The problem can be particularly serious if an adjacent channel user is transmitting in very close range to a subscriber’s receiver, while the receiver attempts to receive a base station on the desired channel. This is referred to as the near–far effect, where a nearby transmitter (which may or may not be of the same type as that used by the cellular system) captures the receiver of the subscriber. Alternatively, the near–far effect occurs when a mobile close to a base station transmits on a channel close to one being used by a weak mobile. The base station may have difficulty in discriminating the desired mobile user from the “bleedover” caused by the close adjacent channel mobile.

Adjacent channel interference can be minimized through careful filtering and channel assignments. Since each cell is given only a fraction of the available channels, a cell need not be assigned channels which are all adjacent in frequency. By keeping the frequency separation between each channel in a given cell as large as possible, the adjacent channel interference may be reduced considerably. Thus instead of assigning channels which form a contiguous band of frequencies within a particular cell, channels are allocated such that the frequency separation between channels in a given cell is maximized. By sequentially assigning successive channels in the frequency band to different cells, many channel allocation schemes are able to separate adjacent channels in a cell by as many as \( N \) channel bandwidths, where \( N \) is the cluster size. Some channel allocation schemes also prevent a secondary source of adjacent channel interference by avoiding the use of adjacent channels in neighboring cell sites.

If the frequency reuse factor is large (e.g., small \( N \)), the separation between adjacent channels at the base station may not be sufficient to keep the adjacent channel interference level within tolerable limits. For example, if a close-in mobile is 20 times as close to the base station as another mobile and has energy spillout of its passband, the signal-to-interference ratio at the base station for the weak mobile (before receiver filtering) is approximately

\[
\frac{S}{I} = (20)^{-n}
\]

(3.12)

For a path loss exponent \( n = 4 \), this is equal to –52 dB. If the intermediate frequency (IF) filter of the base station receiver has a slope of 20 dB/octave, then an adjacent channel interferer must be displaced by at least six times the passband bandwidth from the center of the receiver frequency passband to achieve 52 dB attenuation. Here, a separation of approximately six channel bandwidths is required for typical filters in order to provide 0 dB SIR from a close-in adjacent channel user. This implies more than six channel separations are needed to bring the adjacent channel interference to an acceptable level. Tight base station filters are needed when close-in and distant users share the same cell. In practice, base station receivers are preceded by a high Q cavity filter in order to reject adjacent channel interference.
Example 3.3
This example illustrates how channels are divided into subsets and allocated to different cells so that adjacent channel interference is minimized. The United States AMPS system initially operated with 666 duplex channels. In 1989, the FCC allocated an additional 10 MHz of spectrum for cellular services, and this allowed 166 new channels to be added to the AMPS system. There are now 832 channels used in AMPS. The forward channel (870.030 MHz) along with the corresponding reverse channel (825.030 MHz) is numbered as channel 1. Similarly, the forward channel 889.98 MHz along with the reverse channel 844.98 MHz is numbered as channel 666 (see Figure 1.2). The extended band has channels numbered as 667 through 799, and 990 through 1023.

In order to encourage competition, the FCC licensed the channels to two competing operators in every service area, and each operator received half of the total channels. The channels used by the two operators are distinguished as block A and block B channels. Block B is operated by companies which have traditionally provided telephone services (called wireline operators), and Block A is operated by companies that have not traditionally provided telephone services (called nonwireline operators).

Out of the 416 channels used by each operator, 395 are voice channels and the remaining 21 are control channels. Channels 1 through 312 (voice channels) and channels 313 through 333 (control channels) are block A channels, and channels 355 through 666 (voice channels) and channels 334 through 354 (control channels) are block B channels. Channels 667 through 716 and 991 through 1023 are the extended Block A voice channels, and channels 717 through 799 are extended Block B voice channels.

Each of the 21 control channels are assigned such that one control channel is provided for each bank of trunked voice channels. Thus, control channels are often reused differently, often with more SIR protection, than the traffic generating voice channels. For an AMPS system, a single cell is assigned a single control channel, so that a seven-cell reuse scheme would have seven control channels assigned to seven neighboring cells that form a cluster. Two neighboring clusters would be assigned the remaining 14 control channels. Thus, control channels obey a 21-cell reuse scheme and are assigned over three clusters before being reused, even though the voice channels may be assigned using a seven-cell reuse scheme.

Each of the 395 voice channels are divided into 21 subsets, each containing about 19 channels. In each subset, the closest adjacent channel is 21 channels away. In a seven-cell reuse system, each cell uses three subsets of channels. The three subsets are assigned such that every channel in the cell is assured of being separated from every other channel by at least seven channel spacings. This channel assignment scheme is illustrated in Table 3.2. As seen in Table 3.2, each cell uses channels in the subsets, $iA + iB + iC$, where $i$ is an integer from 1 to 7. The total number of voice
channels in a cell is about 57. The channels listed in the upper half of the chart belong to block A and those in the lower half belong to block B. The shaded set of numbers correspond to the control channels which are standard to all cellular systems in North America.

<table>
<thead>
<tr>
<th>1A</th>
<th>2A</th>
<th>3A</th>
<th>4A</th>
<th>5A</th>
<th>6A</th>
<th>7A</th>
<th>1B</th>
<th>2B</th>
<th>3B</th>
<th>4B</th>
<th>5B</th>
<th>6B</th>
<th>7B</th>
<th>1C</th>
<th>2C</th>
<th>3C</th>
<th>4C</th>
<th>5C</th>
<th>6C</th>
<th>7C</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
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<td>12</td>
<td>13</td>
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<td>36</td>
<td>37</td>
<td>38</td>
<td>39</td>
<td>40</td>
<td>41</td>
<td>42</td>
</tr>
</tbody>
</table>

Table 3.2 AMPS Channel Allocation for A and B Side Carriers

3.5.4 Power Control for Reducing Interference

In practical cellular radio and personal communication systems, the power levels transmitted by every subscriber unit are under constant control by the serving base stations. This is done to ensure that each mobile transmits the smallest power necessary to maintain a good quality link...
on the reverse channel. Power control not only helps prolong battery life for the subscriber unit, but also dramatically reduces the reverse channel S/I in the system. As shown in Chapters 9 and 11, power control is especially important for emerging CDMA spread spectrum systems that allow every user in every cell to share the same radio channel.

### 3.6 Trunking and Grade of Service

Cellular radio systems rely on **trunking** to accommodate a large number of users in a limited radio spectrum. The concept of trunking allows a large number of users to share the relatively small number of channels in a cell by providing access to each user, on demand, from a pool of available channels. In a trunked radio system, each user is allocated a channel on a per call basis, and upon termination of the call, the previously occupied channel is immediately returned to the pool of available channels.

Trunking exploits the statistical behavior of users so that a fixed number of channels or circuits may accommodate a large, random user community. The telephone company uses trunking theory to determine the number of telephone circuits that need to be allocated for office buildings with hundreds of telephones, and this same principle is used in designing cellular radio systems. There is a trade-off between the number of available telephone circuits and the likelihood of a particular user finding that no circuits are available during the peak calling time. As the number of phone lines decreases, it becomes more likely that all circuits will be busy for a particular user. In a trunked mobile radio system, when a particular user requests service and all of the radio channels are already in use, the user is blocked, or denied access to the system. In some systems, a queue may be used to hold the requesting users until a channel becomes available.

To design trunked radio systems that can handle a specific capacity at a specific “grade of service,” it is essential to understand trunking theory and queuing theory. The fundamentals of trunking theory were developed by Erlang, a Danish mathematician who, in the late 19th century, embarked on the study of how a large population could be accommodated by a limited number of servers [Bou88]. Today, the measure of traffic intensity bears his name. One Erlang represents the amount of traffic intensity carried by a channel that is completely occupied (i.e. one call-hour per hour or one call-minute per minute). For example, a radio channel that is occupied for thirty minutes during an hour carries 0.5 Erlangs of traffic.

The **grade of service (GOS)** is a measure of the ability of a user to access a trunked system during the busiest hour. The busy hour is based upon customer demand at the busiest hour during a week, month, or year. The busy hours for cellular radio systems typically occur during rush hours, between 4 p.m. and 6 p.m. on a Thursday or Friday evening. The grade of service is a benchmark used to define the desired performance of a particular trunked system by specifying a desired likelihood of a user obtaining channel access given a specific number of channels available in the system. It is the wireless designer’s job to estimate the maximum required capacity and to allocate the proper number of channels in order to meet the GOS. GOS is typically given as the likelihood that a call is blocked, or the likelihood of a call experiencing a delay greater than a certain queuing time.
A number of definitions listed in Table 3.3 are used in trunking theory to make capacity estimates in trunked systems.

The traffic intensity offered by each user is equal to the call request rate multiplied by the holding time. That is, each user generates a traffic intensity of $A_u$ Erlangs given by

$$A_u = \lambda H$$

where $H$ is the average duration of a call and $\lambda$ is the average number of call requests per unit time for each user. For a system containing $U$ users and an unspecified number of channels, the total offered traffic intensity $A$, is given as

$$A = UA_u$$

Furthermore, in a $C$ channel trunked system, if the traffic is equally distributed among the channels, then the traffic intensity per channel, $A_c$, is given as

$$A_c = UA_u/C$$

Note that the offered traffic is not necessarily the traffic which is carried by the trunked system, only that which is offered to the trunked system. When the offered traffic exceeds the maximum capacity of the system, the carried traffic becomes limited due to the limited capacity (i.e. limited number of channels). The maximum possible carried traffic is the total number of channels, $C$, in Erlangs. The AMPS cellular system is designed for a GOS of 2% blocking. This implies that the channel allocations for cell sites are designed so that 2 out of 100 calls will be blocked due to channel occupancy during the busiest hour.

There are two types of trunked systems which are commonly used. The first type offers no queuing for call requests. That is, for every user who requests service, it is assumed there is no setup time and the user is given immediate access to a channel if one is available. If no channels are available, the requesting user is blocked without access and is free to try again later. This type of trunking is called blocked calls cleared and assumes that calls arrive as determined by a Poisson distribution. Furthermore, it is assumed that there are an infinite number of users as well.
as the following: (a) there are memoryless arrivals of requests, implying that all users, including blocked users, may request a channel at any time; (b) the probability of a user occupying a channel is exponentially distributed, so that longer calls are less likely to occur as described by an exponential distribution; and (c) there are a finite number of channels available in the trunking pool. This is known as an M/M/m/m queue, and leads to the derivation of the Erlang B formula (also known as the blocked calls cleared formula). The Erlang B formula determines the probability that a call is blocked and is a measure of the GOS for a trunked system which provides no queuing for blocked calls. The Erlang B formula is derived in Appendix A and is given by

\[ Pr[\text{blocking}] = \frac{A^C}{C!} = GOS \]  

where \( C \) is the number of trunked channels offered by a trunked radio system and \( A \) is the total offered traffic. While it is possible to model trunked systems with finite users, the resulting expressions are much more complicated than the Erlang B result, and the added complexity is not warranted for typical trunked systems which have users that outnumber available channels by orders of magnitude. Furthermore, the Erlang B formula provides a conservative estimate of the GOS, as the finite user results always predict a smaller likelihood of blocking. The capacity of a trunked radio system where blocked calls are lost is tabulated for various values of GOS and numbers of channels in Table 3.4.

<table>
<thead>
<tr>
<th>Number of Channels ( C )</th>
<th>Capacity (Erlangs) for GOS = 0.01</th>
<th>= 0.005</th>
<th>= 0.002</th>
<th>= 0.001</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.153</td>
<td>0.105</td>
<td>0.065</td>
<td>0.046</td>
</tr>
<tr>
<td>4</td>
<td>0.869</td>
<td>0.701</td>
<td>0.535</td>
<td>0.439</td>
</tr>
<tr>
<td>5</td>
<td>1.36</td>
<td>1.13</td>
<td>0.900</td>
<td>0.762</td>
</tr>
<tr>
<td>10</td>
<td>4.46</td>
<td>3.96</td>
<td>3.43</td>
<td>3.09</td>
</tr>
<tr>
<td>20</td>
<td>12.0</td>
<td>11.1</td>
<td>10.1</td>
<td>9.41</td>
</tr>
<tr>
<td>24</td>
<td>15.3</td>
<td>14.2</td>
<td>13.0</td>
<td>12.2</td>
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<tr>
<td>40</td>
<td>29.0</td>
<td>27.3</td>
<td>25.7</td>
<td>24.5</td>
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<tr>
<td>70</td>
<td>56.1</td>
<td>53.7</td>
<td>51.0</td>
<td>49.2</td>
</tr>
<tr>
<td>100</td>
<td>84.1</td>
<td>80.9</td>
<td>77.4</td>
<td>75.2</td>
</tr>
</tbody>
</table>
The second kind of trunked system is one in which a queue is provided to hold calls which are blocked. If a channel is not available immediately, the call request may be delayed until a channel becomes available. This type of trunking is called Blocked Calls Delayed, and its measure of GOS is defined as the probability that a call is blocked after waiting a specific length of time in the queue. To find the GOS, it is first necessary to find the likelihood that a call is initially denied access to the system. The likelihood of a call not having immediate access to a channel is determined by the Erlang C formula derived in Appendix A:

\[ \text{Pr}[\text{delay} > 0] = \frac{A^C}{A^C + C! \left(1 - \frac{A}{C}\right) \sum_{k=0}^{C-1} \frac{A^k}{k!}} \]  

(3.17)

If no channels are immediately available the call is delayed, and the probability that the delayed call is forced to wait more than \( t \) seconds is given by the probability that a call is delayed, multiplied by the conditional probability that the delay is greater than \( t \) seconds. The GOS of a trunked system where blocked calls are delayed is hence given by:

\[ \text{Pr}[\text{delay} > t] = \text{Pr}[\text{delay} > 0] \text{Pr}[\text{delay} > t|\text{delay} > 0] \]  

(3.18)

The average delay \( D \) for all calls in a queued system is given by:

\[ D = \text{Pr}[\text{delay} > 0] \frac{H}{C - A} \]  

(3.19)

where the average delay for those calls which are queued is given by \( H/(C - A) \).

The Erlang B and Erlang C formulas are plotted in graphical form in Figures 3.6 and 3.7. These graphs are useful for determining GOS in rapid fashion, although computer simulations are often used to determine transient behaviors experienced by particular users in a mobile system.

To use Figures 3.6 and 3.7, locate the number of channels on the top portion of the graph. Locate the traffic intensity of the system on the bottom portion of the graph. The blocking probability \( \text{Pr}[\text{blocking}] \) is shown on the abscissa of Figure 3.6, and \( \text{Pr}[\text{delay}] > 0 \) is shown on the abscissa of Figure 3.7. With two of the parameters specified, it is easy to find the third parameter.

**Example 3.4**

How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system? (a) 1, (b) 5, (c) 10, (d) 20, (e) 100. Assume each user generates 0.1 Erlangs of traffic.

**Solution**

From Table 3.4, we can find the total capacity in Erlangs for the 0.5% GOS for different numbers of channels. By using the relation \( A = UA_n \), we can obtain the total number of users that can be supported in the system.
Figure 3.6  The Erlang B chart showing the probability of blocking as functions of the number of channels and traffic intensity in Erlangs.
Figure 3.7  The Erlang C chart showing the probability of a call being delayed as a function of the number of channels and traffic intensity in Erlangs.
(a) Given \( C = 1, A_u = 0.1, GOS = 0.005 \)

From Figure 3.6, we obtain \( A = 0.005 \).

Therefore, total number of users, \( U = A/A_u = 0.005/0.1 = 0.05 \) users.

But, actually one user could be supported on one channel. So, \( U = 1 \).

(b) Given \( C = 5, A_u = 0.1, GOS = 0.005 \)

From Figure 3.6, we obtain \( A = 1.13 \).

Therefore, total number of users, \( U = A/A_u = 1.13/0.1 = 11 \) users.

(c) Given \( C = 10, A_u = 0.1, GOS = 0.005 \)

From Figure 3.6, we obtain \( A = 3.96 \).

Therefore, total number of users, \( U = A/A_u = 3.96/0.1 = 39 \) users.

(d) Given \( C = 20, A_u = 0.1, GOS = 0.005 \)

From Figure 3.6, we obtain \( A = 11.10 \).

Therefore, total number of users, \( U = A/A_u = 11.10/0.1 = 110 \) users.

(e) Given \( C = 100, A_u = 0.1, GOS = 0.005 \)

From Figure 3.6, we obtain \( A = 80.9 \).

Therefore, total number of users, \( U = A/A_u = 80.9/0.1 = 809 \) users.

Example 3.5

An urban area has a population of two million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages two calls per hour at an average call duration of three minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

Solution

System A

Given:

- Probability of blocking = 2% = 0.02
- Number of channels per cell used in the system, \( C = 19 \)
- Traffic intensity per user, \( A_u = \lambda H = 2 \times (3/60) = 0.1 \) Erlangs

For \( GOS = 0.02 \) and \( C = 19 \), from the Erlang B chart, the total carried traffic, \( A \), is obtained as 12 Erlangs.

Therefore, the number of users that can be supported per cell is

\[ U = A/A_u = 12/0.1 = 120 \]

Since there are 394 cells, the total number of subscribers that can be supported by System A is equal to \( 120 \times 394 = 47280 \)

System B

Given:

- Probability of blocking = 2% = 0.02
- Number of channels per cell used in the system, \( C = 57 \)
- Traffic intensity per user, \( A_u = \lambda H = 2 \times (3/60) = 0.1 \) Erlangs

...
For $GOS = 0.02$ and $C = 57$, from the Erlang B chart, the total carried traffic, $A$, is obtained as 45 Erlangs.

Therefore, the number of users that can be supported per cell is $U = \frac{A}{A_u} = \frac{45}{0.1} = 450$.

Since there are 98 cells, the total number of subscribers that can be supported by System B is equal to $450 \times 98 = 44,100$.

System C

Given:
Probability of blocking = 2% = 0.02
Number of channels per cell used in the system, $C = 100$
Traffic intensity per user, $A_u = \lambda H = 2 \times (3/60) = 0.1$ Erlangs

For $GOS = 0.02$ and $C = 100$, from the Erlang B chart, the total carried traffic, $A$, is obtained as 88 Erlangs.

Therefore, the number of users that can be supported per cell is $U = \frac{A}{A_u} = \frac{88}{0.1} = 880$.

Since there are 49 cells, the total number of subscribers that can be supported by System C is equal to $880 \times 49 = 43,120$.

Therefore, total number of cellular subscribers that can be supported by these three systems are $47,280 + 44,100 + 43,120 = 134,500$ users.

Since there are two million residents in the given urban area and the total number of cellular subscribers in System A is equal to 47280, the percentage market penetration is equal to $\frac{47,280}{2,000,000} = 2.36\%$.

Similarly, market penetration of System B is equal to $\frac{44,100}{2,000,000} = 2.205\%$.

and the market penetration of System C is equal to $\frac{43,120}{2,000,000} = 2.156\%$.

The market penetration of the three systems combined is equal to $\frac{134,500}{2,000,000} = 6.725\%$.

Example 3.6

A certain city has an area of 1,300 square miles and is covered by a cellular system using a seven-cell reuse pattern. Each cell has a radius of four miles and the city is allocated 40 MHz of spectrum with a full duplex channel bandwidth of 60 kHz. Assume a GOS of 2% for an Erlang B system is specified. If the offered traffic per user is 0.03 Erlangs, compute (a) the number of cells in the service area, (b) the number of channels per cell, (c) traffic intensity of each cell, (d) the maximum carried traffic, (e) the total number of users that can be served for 2% GOS, (f) the number of mobiles per unique channel (where it is understood that channels are reused), and (g) the theoretical maximum number of users that could be served at one time by the system.
Trunking and Grade of Service

Solution

(a) Given:
Total coverage area = 1300 miles, and cell radius = 4 miles
The area of a cell (hexagon) can be shown to be $2.5981R^2$, thus each cell covers $2.5981 \times (4)^2 = 41.57$ sq. mi.
Hence, the total number of cells are $N_c = \frac{1300}{41.57} = 31$ cells.

(b) The total number of channels per cell ($C$)
= allocated spectrum / (channel width \times frequency reuse factor)
= $\frac{40,000,000}{(60,000 \times 7)} = 95$ channels/cell

(c) Given:
$C = 95$, and $GOS = 0.02$
From the Erlang B chart, we have
traffic intensity per cell $A = 84$ Erlangs/cell

(d) Maximum carried traffic = number of cells \times traffic intensity per cell
= $31 \times 84 = 2604$ Erlangs.

(e) Given traffic per user = 0.03 Erlangs
Total number of users = Total traffic / traffic per user
= $2604 / 0.03 = 86,800$ users.

(f) Number of mobiles per channel = number of users / number of channels
= $86,800 / 666 = 130$ mobiles/channel.

(g) The theoretical maximum number of served mobiles is the number of available channels in the system (all channels occupied)
= $C \times N_c = 95 \times 31 = 2945$ users, which is $3.4\%$ of the customer base.

Example 3.7
A hexagonal cell within a four-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and $\lambda = 1$ call/hour, compute the following for an Erlang C system that has a 5\% probability of a delayed call:

(a) How many users per square kilometer will this system support?
(b) What is the probability that a delayed call will have to wait for more than 10 s?
(c) What is the probability that a call will be delayed for more than 10 seconds?

Solution

Given:
Cell radius, $R = 1.387$ km
Area covered per cell is $2.598 \times (1.387)^2 = 5$ sq km
Number of cells per cluster = 4
Total number of channels = 60
Therefore, number of channels per cell = $60 / 4 = 15$ channels.

(a) From Erlang C chart, for 5\% probability of delay with $C = 15$, traffic intensity = 9.0 Erlangs.
Therefore, number of users = total traffic intensity / traffic per user
\[= \frac{9.0}{0.029} = 310 \text{ users}\]
\[= 310 \text{ users/sq km} = 62 \text{ users/sq km}\]

(b) Given \(\lambda = 1\), holding time\[H = \frac{A_t}{\lambda} = 0.029 \text{ hour} = 104.4 \text{ seconds}.
\]
The probability that a delayed call will have to wait longer than 10 s is
\[Pr[\text{delay} > t | \text{delay}] = \exp\left(-\frac{(C - A)t}{H}\right)\]
\[= \exp\left(-\frac{(15 - 9.0)10}{104.4}\right) = 56.29\%
\]

(c) Given \(Pr[\text{delay} > 0] = 5\% = 0.05\)
Probability that a call is delayed more than 10 seconds,
\[Pr[\text{delay} > 10] = Pr[\text{delay} > 0]Pr[\text{delay} > t | \text{delay}]\]
\[= 0.05 \times 0.5629 = 2.81\%
\]

Trunking efficiency is a measure of the number of users which can be offered a particular GOS with a particular configuration of fixed channels. The way in which channels are grouped can substantially alter the number of users handled by a trunked system. For example, from Table 3.4, 10 trunked channels at a GOS of 0.01 can support 4.46 Erlangs of traffic, whereas two groups of five trunked channels can support \(2 \times 1.36\) Erlangs, or 2.72 Erlangs of traffic. Clearly, 10 channels trunked together support 60\% more traffic at a specific GOS than do two five channel trunks! It should be clear that the allocation of channels in a trunked radio system has a major impact on overall system capacity.

3.7 Improving Coverage and Capacity in Cellular Systems

As the demand for wireless service increases, the number of channels assigned to a cell eventually becomes insufficient to support the required number of users. At this point, cellular design techniques are needed to provide more channels per unit coverage area. Techniques such as cell splitting, sectoring, and coverage zone approaches are used in practice to expand the capacity of cellular systems. Cell splitting allows an orderly growth of the cellular system. Sectoring uses directional antennas to further control the interference and frequency reuse of channels. The zone microcell concept distributes the coverage of a cell and extends the cell boundary to hard-to-reach places. While cell splitting increases the number of base stations in order to increase capacity, sectoring and zone microcells rely on base station antenna placements to improve capacity by reducing co-channel interference. Cell splitting and zone microcell techniques do not suffer the trunking inefficiencies experienced by sectored cells, and enable the base station to oversee all handoff chores related to the microcells, thus reducing the computational load at the MSC. These three popular capacity improvement techniques will be explained in detail.

3.7.1 Cell Splitting

Cell splitting is the process of subdividing a congested cell into smaller cells, each with its own base station and a corresponding reduction in antenna height and transmitter power. Cell splitting increases the capacity of a cellular system since it increases the number of times that chan-
channels are reused. By defining new cells which have a smaller radius than the original cells and by installing these smaller cells (called microcells) between the existing cells, capacity increases due to the additional number of channels per unit area.

Imagine if every cell in Figure 3.1 were reduced in such a way that the radius of every cell was cut in half. In order to cover the entire service area with smaller cells, approximately four times as many cells would be required. This can be easily shown by considering a circle with radius $R$. The area covered by such a circle is four times as large as the area covered by a circle with radius $R/2$. The increased number of cells would increase the number of clusters over the coverage region, which in turn would increase the number of channels, and thus capacity, in the coverage area. Cell splitting allows a system to grow by replacing large cells with smaller cells, while not upsetting the channel allocation scheme required to maintain the minimum co-channel reuse ratio $Q$ (see Equation (3.4)) between co-channel cells.

An example of cell splitting is shown in Figure 3.8. In Figure 3.8, the base stations are placed at corners of the cells, and the area served by base station $A$ is assumed to be saturated with traffic (i.e., the blocking of base station $A$ exceeds acceptable rates). New base stations are therefore needed in the region to increase the number of channels in the area and to reduce the area served by the single base station. Note in the figure that the original base station $A$ has been surrounded by six new microcells. In the example shown in Figure 3.8, the smaller cells were added in such a way as to preserve the frequency reuse plan of the system. For example, the microcell base station labeled $G$ was placed halfway between two larger stations utilizing the same channel set $G$. This is also the case for the other microcells in the figure. As can be seen from Figure 3.8, cell splitting merely scales the geometry of the cluster. In this case, the radius of each new microcell is half that of the original cell.

![Figure 3.8 Illustration of cell splitting.](image-url)
For the new cells to be smaller in size, the transmit power of these cells must be reduced. The transmit power of the new cells with radius half that of the original cells can be found by examining the received power $P_r$ at the new and old cell boundaries and setting them equal to each other. This is necessary to ensure that the frequency reuse plan for the new microcells behaves exactly as for the original cells. For Figure 3.8

$$P_r[\text{at old cell boundary}] \propto P_t R^{-n} \quad (3.20)$$

and

$$P_r[\text{at new cell boundary}] \propto P_t (R/2)^{-n} \quad (3.21)$$

where $P_{t1}$ and $P_{t2}$ are the transmit powers of the larger and smaller cell base stations, respectively, and $n$ is the path loss exponent. If we take $n = 4$ and set the received powers equal to each other, then

$$P_{t2} = \frac{P_{t1}}{16} \quad (3.22)$$

In other words, the transmit power must be reduced by 12 dB in order to fill in the original coverage area with microcells, while maintaining the $S/I$ requirement.

In practice, not all cells are split at the same time. It is often difficult for service providers to find real estate that is perfectly situated for cell splitting. Therefore, different cell sizes will exist simultaneously. In such situations, special care needs to be taken to keep the distance between co-channel cells at the required minimum, and hence channel assignments become more complicated [Rap97]. Also, handoff issues must be addressed so that high speed and low speed traffic can be simultaneously accommodated (the umbrella cell approach of Section 3.4 is commonly used). When there are two cell sizes in the same region as shown in Figure 3.8, Equation (3.22) shows that one cannot simply use the original transmit power for all new cells or the new transmit power for all the original cells. If the larger transmit power is used for all cells, some channels used by the smaller cells would not be sufficiently separated from co-channel cells. On the other hand, if the smaller transmit power is used for all the cells, there would be parts of the larger cells left unserved. For this reason, channels in the old cell must be broken down into two channel groups, one that corresponds to the smaller cell reuse requirements and the other that corresponds to the larger cell reuse requirements. The larger cell is usually dedicated to high speed traffic so that handoffs occur less frequently.

The two channel group sizes depend on the stage of the splitting process. At the beginning of the cell splitting process, there will be fewer channels in the small power groups. However, as demand grows, more channels will be required, and thus the smaller groups will require more channels. This splitting process continues until all the channels in an area are used in the lower power group, at which point cell splitting is complete within the region, and the entire system is rescaled to have a smaller radius per cell. Antenna downtilting, which deliberately focuses radiated energy from the base station toward the ground (rather than toward the horizon), is often used to limit the radio coverage of newly formed microcells.
Example 3.8
Consider Figure 3.9. Assume each base station uses 60 channels, regardless of cell size. If each original cell has a radius of 1 km and each microcell has a radius of 0.5 km, find the number of channels contained in a 3 km by 3 km square centered around A under the following conditions: (a) without the use of microcells; (b) when the lettered microcells as shown in Figure 3.9 are used; and (c) if all the original base stations are replaced by microcells. Assume cells on the edge of the square to be contained within the square.

Solution
(a) without the use of microcells:
A cell radius of 1 km implies that the sides of the larger hexagons are also 1 km in length. To cover the 3 km by 3 km square centered around base station A, we need to cover 1.5 km (1.5 times the hexagon radius) toward the right, left, top, and bottom of base station A. This is shown in Figure 3.9. From Figure 3.9, we see that this area contains five base stations. Since each base station has 60 channels, the total number of channels without cell splitting is equal to $5 \times 60 = 300$ channels.
(b) with the use of the microcells as shown in Figure 3.9:
In Figure 3.9, the base station A is surrounded by six microcells. Therefore, the total number of base stations in the square area under study is equal to $5 + 6 = 11$. Since each base station has 60 channels, the total number of channels will be equal to $11 \times 60 = 660$ channels. This is a 2.2 times increase in capacity when compared to case (a).

(c) if all the base stations are replaced by microcells:
From Figure 3.9, we see there are a total of $5 + 12 = 17$ base stations in the square region under study. Since each base station has 60 channels, the total number of channels will be equal to $17 \times 60 = 1020$ channels. This is a 3.4 times increase in capacity compared to case (a).

Theoretically, if all cells were microcells having half the radius of the original cell, the capacity increase would approach four.

### 3.7.2 Sectoring
As shown in Section 3.7.1, cell splitting achieves capacity improvement by essentially rescaling the system. By decreasing the cell radius $R$ and keeping the co-channel reuse ratio $D/R$ unchanged, cell splitting increases the number of channels per unit area.

However, another way to increase capacity is to keep the cell radius unchanged and seek methods to decrease the $D/R$ ratio. As we now show, sectoring increases SIR so that the cluster size may be reduced. In this approach, first the SIR is improved using directional antennas, then capacity improvement is achieved by reducing the number of cells in a cluster, thus increasing the frequency reuse. However, in order to do this successfully, it is necessary to reduce the relative interference without decreasing the transmit power.

The co-channel interference in a cellular system may be decreased by replacing a single omnidirectional antenna at the base station by several directional antennas, each radiating within a specified sector. By using directional antennas, a given cell will receive interference and transmit with only a fraction of the available co-channel cells. The technique for decreasing co-channel interference and thus increasing system performance by using directional antennas is called sectoring. The factor by which the co-channel interference is reduced depends on the amount of sectoring used. A cell is normally partitioned into three $120^\circ$ sectors or six $60^\circ$ sectors as shown in Figure 3.10(a) and (b).

![Figure 3.10](image)

**Figure 3.10** (a) 120° sectoring; (b) 60° sectoring.
When sectoring is employed, the channels used in a particular cell are broken down into sectored groups and are used only within a particular sector, as illustrated in Figure 3.10(a) and (b). Assuming seven-cell reuse, for the case of 120° sectors, the number of interferers in the first tier is reduced from six to two. This is because only two of the six co-channel cells receive interference with a particular sectored channel group. Referring to Figure 3.11, consider the interference experienced by a mobile located in the right-most sector in the center cell labeled “5” to the right of the center cell, and three to the left of the center cell. Out of these six co-channel cells, only two cells have sectors with antenna patterns which radiate into the center cell, and hence a mobile in the center cell will experience interference on the forward link from only these two sectors. The resulting $S/I$ for this case can be found using Equation (3.8) to be 24.2 dB, which is a significant improvement over the omnidirectional case in Section 3.5, where the worst case $S/I$ was shown to be 17 dB. This $S/I$ improvement allows the wireless engineer to then decrease the cluster size $N$ in order to improve the frequency reuse, and thus the system capacity. In practical systems, further improvement in $S/I$ is achieved by downtilting the sector antennas such that the radiation pattern in the vertical (elevation) plane has a notch at the nearest co-channel cell distance.

![Figure 3.11](image_url)

**Figure 3.11** Illustration of how 120° sectoring reduces interference from co-channel cells. Out of the 6 co-channel cells in the first tier, only two of them interfere with the center cell. If omnidirectional antennas were used at each base station, all six co-channel cells would interfere with the center cell.
The improvement in $S/I$ implies that with $120^\circ$ sectoring, the minimum required $S/I$ of 18 dB can be easily achieved with seven-cell reuse, as compared to 12-cell reuse for the worst possible situation in the unsectored case (see Section 3.5.1). Thus, sectoring reduces interference, which amounts to an increase in capacity by a factor of $12/7$, or 1.714. In practice, the reduction in interference offered by sectoring enable planners to reduce the cluster size $N$, and provides an additional degree of freedom in assigning channels. The penalty for improved $S/I$ and the resulting capacity improvement from the shrinking cluster size is an increased number of antennas at each base station, and a decrease in trunking efficiency due to channel sectoring at the base station. Since sectoring reduces the coverage area of a particular group of channels, the number of handoffs increases, as well. Fortunately, many modern base stations support sectorization and allow mobiles to be handed off from sector to sector within the same cell without intervention from the MSC, so the handoff problem is often not a major concern.

It is the loss of traffic due to decreased trunking efficiency that causes some operators to shy away from the sectoring approach, particularly in dense urban areas where the directional antenna patterns are somewhat ineffective in controlling radio propagation. Because sectoring uses more than one antenna per base station, the available channels in the cell must be subdivided and dedicated to a specific antenna. This breaks up the available trunked channel pool into several smaller pools, and decreases trunking efficiency.

### Example 3.9
Consider a cellular system in which an average call lasts two minutes, and the probability of blocking is to be no more than 1%. Assume that every subscriber makes one call per hour, on average. If there are a total of 395 traffic channels for a seven-cell reuse system, there will be about 57 traffic channels per cell. Assume that blocked calls are cleared so the blocking is described by the Erlang B distribution. From the Erlang B distribution, it can be found that the unsectored system may handle 44.2 Erlangs or 1326 calls per hour.

Now employing $120^\circ$ sectoring, there are only 19 channels per antenna sector (57/3 antennas). For the same probability of blocking and average call length, it can be found from the Erlang B distribution that each sector can handle 11.2 Erlangs or 336 calls per hour. Since each cell consists of three sectors, this provides a cell capacity of $3 \times 336 = 1008$ calls per hour, which amounts to a 24% decrease when compared to the unsectored case. Thus, sectoring decreases the trunking efficiency while improving the $S/I$ for each user in the system.

It can be found that using $60^\circ$ sectors improves the $S/I$ even more. In this case, the number of first tier interferers is reduced from six to only one. This results in $S/I = 29$ dB for a seven-cell system and enables four-cell reuse. Of course, using six sectors per cell reduces the trunking efficiency and increases the number of necessary handoffs even more. If the unsectored system is compared to the six sector case, the degradation in trunking efficiency can be shown to be 44%. (The proof of this is left as an exercise.)
3.7.3 Repeaters for Range Extension

Often a wireless operator needs to provide dedicated coverage for hard-to-reach areas, such as within buildings, or in valleys or tunnels. Radio retransmitters, known as repeaters, are often used to provide such range extension capabilities. Repeaters are bidirectional in nature, and simultaneously send signals to and receive signals from a serving base station. Repeaters work using over-the-air signals, so they may be installed anywhere and are capable of repeating an entire cellular or PCS band. Upon receiving signals from a base station forward link, the repeater amplifies and reradiates the base station signals to the specific coverage region. Unfortunately, the received noise and interference is also reradiated by the repeater on both the forward and reverse link, so care must be taken to properly place the repeaters, and to adjust the various forward and reverse link amplifier levels and antenna patterns. Repeaters can be easily thought of as bidirectional “bent pipes” that retransmit what has been received. In practice, directional antennas or distributed antenna systems (DAS) are connected to the inputs or outputs of repeaters for localized spot coverage, particularly in tunnels or buildings.

By modifying the coverage of a serving cell, an operator is able to dedicate a certain amount of the base station’s traffic for the areas covered by the repeater. However, the repeater does not add capacity to the system—it simply serves to reradiate the base station signal into specific locations. Repeaters are increasingly being used to provide coverage into and around buildings, where coverage has been traditionally weak [Rap96], [Mor00]. Many carriers have opted to provide in-building wireless penetration by installing microcells outside of large buildings, and then installing many repeaters with DAS networks within the buildings. This approach provides immediate coverage into targeted areas, but does not accommodate the increases in capacity that will arise due to increased outdoor and indoor user traffic. Eventually, dedicated base stations within buildings will be needed to accommodate the large number of in-building cellular users.

Determining the proper location for repeaters and distributed antenna systems within buildings requires careful planning, particularly due to the fact that interference levels are reradiated into the building from the base station and from the interior of the building back to the base station. Also, repeaters must be provisioned to match the available capacity from the serving base station. Fortunately, software products, such as SitePlanner [Wir01], allow engineers to rapidly determine the best placements for repeaters and the required DAS network while simultaneously computing the available traffic and associated cost of the installation. SitePlanner is protected by US Patent 6,317,599 and other patents. Using SitePlanner, engineers can very quickly determine the proper provisioning for a particular level of range extension (see Figure 3.12).

3.7.4 A Microcell Zone Concept

The increased number of handoffs required when sectoring is employed results in an increased load on the switching and control link elements of the mobile system. A solution to this problem was presented by Lee [Lee91b]. This proposal is based on a microcell concept for seven cell reuse, as illustrated in Figure 3.13. In this scheme, each of the three (or possibly more) zone sites
(represented as Tx/Rx in Figure 3.13) are connected to a single base station and share the same radio equipment. The zones are connected by coaxial cable, fiber optic cable, or microwave link to the base station. Multiple zones and a single base station make up a cell. As a mobile travels within the cell, it is served by the zone with the strongest signal. This approach is superior to sectoring since antennas are placed at the outer edges of the cell, and any base station channel may be assigned to any zone by the base station.

As a mobile travels from one zone to another within the cell, it retains the same channel. Thus, unlike in sectoring, a handoff is not required at the MSC when the mobile travels between zones within the cell. The base station simply switches the channel to a different zone site. In this way, a given channel is active only in the particular zone in which the mobile is traveling, and hence the base station radiation is localized and interference is reduced. The channels are distributed in time and space by all three zones and are also reused in co-channel cells in the normal fashion. This technique is particularly useful along highways or along urban traffic corridors.

The advantage of the zone cell technique is that while the cell maintains a particular coverage radius, the co-channel interference in the cellular system is reduced since a large central base station is replaced by several lower powered transmitters (zone transmitters) on the edges of the cell. Decreased co-channel interference improves the signal quality and also leads to an increase in capacity without the degradation in trunking efficiency caused by sectoring. As mentioned earlier, an $S/I$ of 18 dB is typically required for satisfactory system performance in narrowband FM. For a system with $N = 7$, a $D/R$ of 4.6 was shown to achieve this. With respect to
Improving Coverage and Capacity in Cellular Systems

The zone microcell system, since transmission at any instant is confined to a particular zone, this implies that a $D_z / R_z$ of 4.6 (where $D_z$ is the minimum distance between active co-channel zones and $R_z$ is the zone radius) can achieve the required link performance. In Figure 3.14, let each individual hexagon represents a zone, while each group of three hexagons represents a cell. The zone radius $R_z$ is approximately equal to one hexagon radius. Now, the capacity of the zone microcell system is directly related to the distance between co-channel cells, and not zones. This distance is represented as $D$ in Figure 3.14. For a $D_z / R_z$ value of 4.6, it can be seen from the geometry of Figure 3.14 that the value of co-channel reuse ratio, $D/R$, is equal to three, where $R$ is the radius of the cell and is equal to twice the length of the hexagon radius. Using Equation (3.4), $D/R = 3$ corresponds to a cluster size of $N = 3$. This reduction in the cluster size from $N = 7$ to $N = 3$ amounts to a 2.33 times increase in capacity for a system completely based on the zone microcell concept. Hence for the same $S/I$ requirement of 18 dB, this system provides a significant increase in capacity over conventional cellular planning.

By examining Figure 3.14 and using Equation (3.8) [Lee91b], the exact worst case $S/I$ of the zone microcell system can be estimated to be 20 dB. Thus, in the worst case, the system provides a margin of 2 dB over the required signal-to-interference ratio while increasing the capacity by 2.33 times over a conventional seven-cell system using omnidirectional antennas.
No loss in trunking efficiency is experienced. Zone cell architectures are being adopted in many cellular and personal communication systems.

### 3.8 Summary

In this chapter, the fundamental concepts of handoff, frequency reuse, trunking efficiency, and frequency planning have been presented. Handoffs are required to pass mobile traffic from cell to cell, and there are various ways handoffs are implemented. The capacity of a cellular system is a function of many variables. The $S/I$ in a wireless propagation channel, along with the specific performance of the air interface in an interference environment, limits the frequency reuse factor of a system, which limits the number of channels within the coverage area. The trunking efficiency limits the number of users that can access a trunked radio system. Trunking is affected by the number of available channels and how they are partitioned in a trunked cellular system. Trunking efficiency is quantified by the GOS. Finally, cell splitting, sectoring, and the zone microcell technique are all shown to improve capacity by increasing $S/I$ in some fashion. The overriding objective in all of these methods is to increase the number of users within the system. The radio propagation characteristics influence the effectiveness of all of these methods in an actual system. Radio propagation is the subject of the following two chapters.

*Figure 3.14* Define $D$, $D_z$, $R$, and $R_z$ for a microcell architecture with $N = 7$. The smaller hexagons form zones and three hexagons (outlined in bold) together form a cell. Six nearest co-channel cells are shown.
3.9 Problems

3.1 Prove that for a hexagonal geometry, the co-channel reuse ratio is given by \( Q = \sqrt{3N} \), where \( N = i^2 + ij + j^2 \). Hint: Use the cosine law and the hexagonal cell geometry.

3.2 If two independent voltage signals, \( v_1(t) \) and \( v_2(t) \), are added together to provide a new resulting signal, prove that under certain conditions the resulting signal has the same power as the sum of the individual powers. What are these conditions? What special conditions must apply for this result to be valid when the signals are uncorrelated?

3.3 Show that the frequency reuse factor for a cellular system is given by \( k/S \), where \( k \) is the average number of channels per cell and \( S \) is the total number of channels available to the cellular service provider.

3.4 If 20 MHz of total spectrum is allocated for a duplex wireless cellular system and each simplex channel has 25 kHz RF bandwidth, find:
   (a) the number of duplex channels.
   (b) the total number of channels per cell site, if \( N = 4 \) cell reuse is used.

3.5 A cellular service provider decides to use a digital TDMA scheme which can tolerate a signal-to-interference ratio of 15 dB in the worst case. Find the optimal value of \( N \) for (a) omnidirectional antennas, (b) 120° sectoring, and (c) 60° sectoring. Should sectoring be used? If so, which case (60° or 120°) should be used? (Assume a path loss exponent of \( n = 4 \) and consider trunking efficiency.)

3.6 You are asked to determine the signal-to-interference ratio (SIR or C/I) on the forward link of a cellular system, when the mobile is located on the fringe of its serving cell. Assume that all cells have equal radii, and that base stations have equal power and are located in the centers of each cell. Also assume that each cell transmits an independent signal, such that interfering signal powers may be added. Let us define a “tier” of cells as being the collection of co-channel cells that are more-or-less the same distance away from the mobile in the serving cell. This problem explores the impact of the cluster size (i.e., frequency reuse distance), the number of tiers used in the calculation of C/I and the effect of the propagation path loss exponent on C/I.

   (a) What is the average distance (in terms of \( R \)) between the mobile on the fringe of the serving cell and the first tier of co-channel cells? (These cells are called the “nearest neighbors.”) How many cells are located in the first tier? Solve for the case of \( N = 1 \), \( N = 3 \), \( N = 4 \), \( N = 7 \), and \( N = 12 \) cluster sizes. How does the average distance compare to the value of \( D = QR \), where \( Q = \sqrt{3N} \) ?

   (b) What is the average distance (in terms of \( R \)) between the mobile on the fringe of the serving cell and the second and third tier of co-channel cells, and how many cells are in the second and third tier of co-channel cells for the cases of \( N = 1 \), \( N = 3 \), \( N = 4 \), \( N = 7 \), and \( N = 12 \) cluster sizes?

   (c) Determine the forward link C/I for the following frequency reuse designs: \( N = 1 \), \( N = 3 \), \( N = 4 \), \( N = 7 \), and \( N = 12 \). Assume that the propagation path loss exponent is four, and evaluate the S/I contribution due to just the first tier and then due to additional outer tiers of co-channel cells. Indicate the number of tiers at which there is a diminishing contribution to the interference at the mobile.

   (d) Repeat part (c), except now consider a line-of-sight path loss exponent of \( n = 2 \). Notice the huge impact that the propagation path loss exponent has on C/I. What can you say...
3.7 Suppose that a mobile station is moving along a straight line between base stations BS$_1$ and BS$_2$, as shown in Figure P3.7. The distance between the base stations is $D = 2000$ m. For simplicity, assume small scale fading is neglected and the received power (in dBm) at base station $i$, from the mobile station, is modeled as a function of distance on the reverse link

$$P_{r,i}(d_i) = P_0 - 10n \log_{10}(d_i/d_0) \text{ (dBm)} \quad i = 1,2$$

where $d_i$ is the distance between the mobile and the base station $i$, in meters. $P_0$ is the received power at distance $d_0$ from the mobile antenna. Assume that $P_0 = 0$ dBm and $d_0 = 1$ m. Let $n$ denote the path loss which is assumed to be equal to 2.9.

Assume the minimum usable signal level for acceptable voice quality at the base station receiver is $P_{r,\text{min}} = -88$ dBm, and the threshold level used by the switch for handoff initiation is $P_{r,\text{HO}}$. Consider that the mobile is currently connected to BS$_1$ and is moving toward a handoff (time required to complete a handoff, once that received signal level reaches the handoff threshold $P_{r,\text{HO}}$ is $\Delta t = 4.5$ seconds).

(a) Determine the minimum required margin $\Delta = P_{r,\text{HO}} - P_{r,\text{min}}$ to assure that calls are not lost due to weak signal condition during handoff. Assume that the base station antenna heights are negligible compared to the distance between the mobile and the base stations.

(b) Describe the effects of the margin $\Delta = P_{r,\text{HO}} - P_{r,\text{min}}$ on the performance of cellular systems.

3.8 If an intensive propagation measurement campaign showed that the mobile radio channel provided a propagation path loss exponent of $n = 3$ instead of four, how would your design decisions in Problem 3.5 change? What is the optimal value of $N$ for the case of $n = 3$?

3.9 Consider a cellular radio system with hexagonal cells and cluster size $N$. Since a hexagonal shape is assumed, the number of co-channel cells in the $i^{th}$ tier of co-channel cells is $6t$, regardless of the cluster size. Considering the forward link, the total co-channel interference power level $I$ received at a particular mobile can be modeled as the sum $\sum_{i=1}^{M} I_i$, where $I_i$ is the interference caused by the $i^{th}$ base station.

Let us suppose that only the base stations in the first three tiers produce significant interference. Interference signals from base stations in more distant tiers are assumed to be negligible.
Problems 99

(a) Assuming that a mobile is located at the boundary of the cell (worst case situation), compute the contribution of co-channel base stations in each tier to the total co-channel interference received at the mobile.

Also, compute the signal-to-interference ratio (SIR) at the mobile when only the first $T$ tiers are considered, with $T = 1, 2, \text{ and } 3$ (all tiers).

Assume that the power received at distance $d$ from the transmitting antenna is given by

$$P_r = P_t \left( \frac{1}{d} \right)^n,$$

where $P_t$ is the transmitted power and $n$ is the path loss exponent. Also, assume that:

- the mobile and base stations are equipped with omnidirectional antennas,
- all base stations are located at the center of the cells and transmit the same power level, and
- all cells have the same radius ($R$).

Present the results for cluster size $N = 1, 3, 4, \text{ and } 7$ and path loss exponents $n = 2, 3, \text{ and } 4$.

(b) Suppose that you are asked to analyze co-channel interference on the forward link of a cellular system with the same characteristics as the system in part (a). In order to reduce the complexity of the analysis (or the computation time, if your analysis is based on simulation), you want to consider as few as possible tiers of co-channel base stations. On the other hand, you want to obtain accurate results from your analysis. Based on the results of part (a), determine the number of tiers that you would use in your co-channel interference analysis for cluster sizes $N = 1, 3, 4, \text{ and } 7$, and path loss exponents $n = 2, 3, \text{ and } 4$.

Assume that you can tolerate an error of 0.5 dB in the computation of SIR with respect to the true value of SIR. Explain and justify your decision.

3.10 A total of 24 MHz of bandwidth is allocated to a particular FDD cellular telephone system that uses two 30 kHz simplex channels to provide full duplex voice and control channels. Assume each cell phone user generates 0.1 Erlangs of traffic. Assume Erlang B is used.

(a) Find the number of channels in each cell for a four-cell reuse system.

(b) If each cell is to offer capacity that is 90% of perfect scheduling, find the maximum number of users that can be supported per cell where omnidirectional antennas are used at each base station.

(c) What is the blocking probability of the system in (b) when the maximum number of users are available in the user pool?

(d) If each new cell now uses 120° sectoring instead of omnidirectional for each base station, what is the new total number of users that can be supported per cell for the same blocking probability as in (c)?

(e) If each cell covers five square kilometers, then how many subscribers could be supported in an urban market that is 50 km $\times$ 50 km for the case of omnidirectional base station antennas?

(f) If each cell covers five square kilometers, then how many subscribers could be supported in an urban market that is 50 km $\times$ 50 km for the case of 120° sectored antennas?

3.11 For a $N = 7$ system with a $Pr[\text{Blocking}] = 1\%$ and average call length of two minutes, find the traffic capacity loss due to trunking for 57 channels when going from omnidirectional antennas to 60° sectored antennas. (Assume that blocked calls are cleared and the average per user call rate is $\lambda = 1$ per hour.)
3.12 Assume that a cell named “Radio Knob” has 57 channels, each with an effective radiated power of 32 W and a cell radius of 10 km. The path loss is 40 dB per decade. The grade of service is established to be a probability of blocking of 5% (assuming blocked calls are cleared). Assume the average call length is two minutes, and each user averages two calls per hour. Further, assume the cell has just reached its maximum capacity and must be split into four new microcells to provide four times the capacity in the same area. (a) What is the current capacity of the “Radio Knob” cell? (b) What is the radius and transmit power of the new cells? (c) How many channels are needed in each of the new cells to maintain frequency reuse stability in the system? (d) If traffic is uniformly distributed, what is the new traffic carried by each new cell? Will the probability of blocking in these new cells be below 0.1% after the split? Assume 57 channels are used at the original base station and the split cells.

3.13 A certain area is covered by a cellular radio system with 84 cells and a cluster size \( N \). 300 voice channels are available for the system. Users are uniformly distributed over the area covered by the cellular system, and the offered traffic per user is 0.04 Erlang. Assume that blocked calls are cleared and the designated blocking probability is \( P_b = 1\% \).

(a) Determine the maximum carried traffic per cell if cluster size \( N = 4 \) is used. Repeat for cluster sizes \( N = 7 \) and 12.

(b) Determine the maximum number of users that can be served by the system for a blocking probability of 1% and cluster size \( N = 4 \). Repeat for cluster sizes \( N = 7 \) and 12.

3.14 The offered traffic in a cellular communication system can be specified by the average call duration \( H \) (also known as holding time) and call request rate \( \lambda \). The quantities \( H \) and \( \lambda \) fairly represent the offered traffic in a particular cell if we assume that mobiles are not crossing cell boundaries. Quantitatively describe the effects of mobile stations that cross cell boundaries on both average call duration and call request rate.

3.15 Exercises in trunking (queueing) theory:

(a) What is the maximum system capacity (total and per channel) in Erlangs when providing a 2% blocking probability with four channels, with 20 channels, with 40 channels?

(b) How many users can be supported with 40 channels at 2% blocking? Assume \( H = 105 \) s, \( \lambda = 1 \) call/hour.

(c) Using the traffic intensity calculated in part (a), find the grade of service in a lost call delayed system for the case of delays being greater than 20 seconds. Assume that \( H = 105 \) s, and determine the GOS for four channels, for 20 channels, for 40 channels.

(d) Comparing part (a) and part (c), does a lost call delayed system with a 20 second queue perform better than a system that clears blocked calls?

3.16 A receiver in an urban cellular radio system detects a 1 mW signal at \( d = d_0 = 1 \) meter from the transmitter. In order to mitigate co-channel interference effects, it is required that the signal received at any base station receiver from another base station transmitter which operates with the same channel must be below \(-100 \) dBm. A measurement team has determined that the average path loss exponent in the system is \( n = 3 \). Determine the major radius of each cell if a seven-cell reuse pattern is used. What is the major radius if a four-cell reuse pattern is used?
3.17 A cellular system using a cluster size of seven is described in Problem 3.16. It is operated with 660 channels, 30 of which are designated as setup (control) channels so that there are about 90 voice channels available per cell. If there is a potential user density of 9000 users/km² in the system, and each user makes an average of one call per hour and each call lasts 1 minute during peak hours, determine the probability that a user will experience a delay greater than 20 seconds if all calls are queued.

3.18 Show that if \( n = 4 \), a cell can be split into four smaller cells, each with half the radius and 1/16 of the transmitter power of the original cell. If extensive measurements show that the path loss exponent is three, how should the transmitter power be changed in order to split a cell into four smaller cells? What impact will this have on the cellular geometry? Explain your answer and provide drawings that show how the new cells would fit within the original macrocells. For simplicity use omnidirectional antennas.

3.19 Using the frequency assignment chart in Table 3.2, design a channelization scheme for a B-side carrier that uses four-cell reuse and three sectors per cell. Include an allocation scheme for the 21 control channels.

3.20 Repeat Problem 3.19 for the case of four-cell reuse and six sectors per cell.

3.21 In practical cellular radio systems, the MSC is programmed to allocate radio channels differently over time for the closest co-channel cells. This technique, called a *hunting sequence*, ensures that co-channel cells first use different channels from within the co-channel set before the same channels are assigned to calls in nearby cells. This minimizes co-channel interference when the cellular system is not fully loaded. Consider three adjoining clusters, and design an algorithm that may be used by the MSC to hunt for appropriate channels when requested from co-channel cells. Assume a seven-cell reuse pattern with three sectors per cell, and use the U.S. cellular channel allocation scheme for the A-side carrier.

3.22 Determine the noise floor (in dBm) for mobile receivers which implement the following standards: (a) AMPS, (b) GSM, (c) USDC, (d) DECT, (e) IS-95, and (f) CT2. Assume all receivers have a noise figure of 10 dB.

3.23 If a base station provides a signal level of –90 dBm at the cell fringe, find the SNR for each of the mobile receivers described in Problem 3.22.

3.24 From first principles, derive the expression for Erlang B given in this chapter.

3.25 Carefully analyze the tradeoff between sectoring and trunking efficiency for a four-cell cluster size. While sectoring improves capacity by improving SIR, there is a loss due to decreased trunking efficiency, since each sector must be trunked separately. Consider a wide range of total available channels per cell and consider the impact of using three sectors and six sectors per cell. Your analysis may involve computer simulation and should indicate the “break even” point when sectoring is not practical.

3.26 Assume each user of a single base station mobile radio system averages three calls per hour, each call lasting an average of 5 minutes.

(a) What is the traffic intensity for each user?

(b) Find the number of users that could use the system with 1% blocking if only one channel is available.
(c) Find the number of users that could use the system with 1% blocking if five trunked channels are available.

(d) If the number of users you found in (c) is suddenly doubled, what is the new blocking probability of the five channel trunked mobile radio system? Would this be acceptable performance? Justify why or why not.

3.27 The U.S. AMPS system is allocated 50 MHz of spectrum in the 800 MHz range and provides 832 channels. Forty-two of those channels are control channels. The forward channel frequency is exactly 45 MHz greater than the reverse channel frequency.

(a) Is the AMPS system simplex, half-duplex, or duplex? What is the bandwidth for each channel and how is it distributed between the base station and the subscriber?

(b) Assume a base station transmits control information on channel 352, operating at 880.560 MHz. What is the transmission frequency of a subscriber unit transmitting on channel 352?

(c) The A-side and B-side cellular carriers evenly split the AMPS channels. Find the number of voice channels and number of control channels for each carrier.

(d) Let’s suppose you are chief engineer of a cellular carrier using seven-cell reuse. Propose a channel assignment strategy for a uniform distribution of users throughout your cellular system. Specifically, assume that each cell has three control channels (120° sectoring is employed) and specify the number of voice channels you would assign to each control channel in your system.

(e) For an ideal hexagonal cellular layout which has identical cell coverage, what is the distance between the centers of two nearest co-channel cells for seven-cell reuse? For four-cell reuse?

3.28 Suppose that you work with a cellular service provider and the cellular radio system your company deployed in a given service area just reached its maximum system capacity. Your boss then asks you to carry out a study to analyze the application of cluster size reduction technique combined with sectoring, aiming to increase the carried traffic of the system. The current deployed system employs the AMPS system, with 300 voice channels, cluster size \( N = 7 \), and omnidirectional antennas at the base stations. Base stations are located at the center of the cells and transmit the same power on the forward link. The designed blocking probability is 2% and all voice channels have been used. The maximum carried traffic per cell is, therefore, 32.8 Erlangs. The minimum SIR (worst case) on the forward link can be computed using the expression

\[
SIR = 10 \log_{10} \left( \frac{\sqrt{3N}}{i_0} \right) \text{ (in dB)},
\]

where \( n \) is the path loss exponent, \( N \) is the cluster size, and \( i_0 \) is the number of interfering base stations in the first tier. For cluster size \( N = 7 \), omnidirectional base station antennas \( (i_0 = 6) \), and \( n = 4 \), we find that \( SIR = 18.7 \text{ dB} \).

When the cluster size is reduced, the maximum carried traffic per cell increases, at the expense of SIR degradation. Sectorized base station antennas can then be used in order to increase SIR and guarantee that the link quality is maintained. In other words, the minimum SIR achieved when cluster size is reduced and sectorized antennas are used must be equal to or exceed the minimum SIR achieved in the current deployed system \( (SIR = 18.7 \text{ dB}) \).

In your analysis of cluster size reduction and sectoring, consider forward link only. Two sectorized antennas are available: \( BW = 60^\circ \) for six sectors per cell, and \( BW = 120^\circ \) for three sectors per cell, as shown in Figure P3.28a. Assume that all cells have hexagonal shape, with radius \( R \).
We want to determine the maximum carried traffic per cell when cluster size is reduced to \( N = 3 \) and \( N = 4 \), using three sectors (\( BW = 60^\circ \)) and six sectors (\( BW = 120^\circ \)) (four possible configurations). Assume that all sectorized antennas are installed at the same time. Consider the first tier of co-channel cells only.

(a) Determine the minimum SIR at the mobile (i.e., when the mobile is located at the cell boundary, as indicated in Figure P3.28b), for cluster sizes \( N = 3 \) and \( N = 4 \), with three and six sectors. Determine which configurations (cluster size \( N \), number of sectors) are feasible regarding co-channel interference (i.e., configurations where the minimum SIR is equal to or exceeds 18.7 dB). Note that the number of interferers in the first tier depends on
the cluster size used and the number of sectors per cell. Use the expression above to compute the minimum \( SIR \).

(b) For each configuration \( (N = 3, 4 \text{ and } 6 \text{ sectors per cell}) \), determine the maximum carried traffic per cell at blocking probability of 2\% and 300 voice channels available in the system. Assume that users are uniformly distributed over the service area and, therefore, all sectors are assigned an equal number of channels.

3.29 Pretend your company won a license to build a U.S. cellular system (the application cost for the license was only $500!). Your license is to cover 140 square km. Assume a base station costs $500,000 and a MTSO costs $1,500,000. An extra $500,000 is needed to advertise and start the business. You have convinced the bank to loan you $6 million, with the idea that in four years you will have earned $10 million in gross billing revenues, and will have paid off the loan.

(a) How many base stations (i.e., cell sites) will you be able to install for $6 million?

(b) Assuming the earth is flat and subscribers are uniformly distributed on the ground, what assumption can you make about the coverage area of each of your cell sites? What is the major radius of each of your cells, assuming a hexagonal mosaic?

(c) Assume that the average customer will pay $50 per month over a four year period. Assume that on the first day you turn your system on, you have a certain number of customers which remains fixed throughout the year. On the first day of each new year, the number of customers using your system doubles and then remains fixed for the rest of that year. What is the minimum number of customers you must have on the first day of service in order to have earned $10 million in gross billing revenues by the end of the 4th year of operation?

(d) For your answer in (c), how many users per square km are needed on the first day of service in order to reach the $10 million mark after the 4th year?